DECIDING ON PARAMETER VALUES OF ANCHOR AND ADJUST HEURISTIC IN STOCK MANAGEMENT

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ABSTRACT

DECIDING ON PARAMETER VALUES OF ANCHOR AND ADJUST HEURISTIC IN STOCK MANAGEMENT

Anchor-and-adjust is a widely used heuristic in stock management because, it is a fine representation of human decision making process in managing a stock. This thesis focuses on two issues related to the usage of the anchor-and-adjust heuristic in stock management: One is the selection of the decision making parameter values and the other is the determination of the desired supply line values for multi-supplier systems.

Weight of Supply Line and *Stock Adjustment Time* are two of the decision parameters of anchor-and-adjust heuristic. We seek a rule of thumb for assigning good values to them. We first introduce a new parameter that we call *Relative Aggressiveness*, which together with *Weight of Supply Line*; determine the nature of the stock behavior. *Relative Aggressiveness* is *Acquisition Delay Time* (delay duration) divided by *Stock Adjustment Time*. We propose 4 as a sufficiently good and applicable value for *Relative Aggressiveness*. In other words, we suggest taking *Stock Adjustment Time* as a quarter of *Acquisition Delay Time*. We also give optimal values of *Weight of Supply Line* in a table for different delay orders and *Relative Aggressiveness* values.

Desired Supply Line is the product of expected acquisition lag and desired acquisition rate. This calculation ensures that supply line would produce the desired acquisition rate given that it is at this desired level. A wrongly calculated *Desired Supply Line* value leads to a steady-state error preventing stock approach its goal. Therefore, correct calculation of *Desired Supply Line* values is crucial. Desired acquisition rate is equal to the expected loss flow in a single-supplier system. However, it is not easy to decide on the desired acquisition rates for a multi-supplier system. We give a general formula for the calculation of *Desired Supply Line* values based on the supplier utilization priorities and supplier production/shipment capacities.

ÖZET

STOK YÖNETİMİNDE ÇAPA VE AYAR SEZGİSELİNİN PARAMETRE DEĞERLERİNİ BELİRLEME

Çapa-ve-ayar, stok yönetiminde insanların düşünce şeklini başarılı bir şekilde yansıttığı için sıklıkla kullanılan bir sezgiseldir. Bu tez çapa-ve-ayar sezgiselindeki iki önemli problem üzerinde yoğunlaşmıştır. Bunlardan biri karar parametrelerinin değerlerinin seçimi, diğeri ise çok tedarikçili sistemlerde ideal tedarik hattı değerinin belirlenmesidir.

Tedarik Hattı Ağırlığı ve *Stok Ayarlama Süresi*, çapa-ve-ayar sezgiselinin iki önemli parametresidir. Biz bu parametrelere uygun değerler atamak için pratik bir yöntem bulmaya çalıştık. Öncelikle, *Tedarik Hattı Ağırlığı* ile birlikte stok davranışını belirleyen, *Göreceli Agresiflik* diye isimlendirdiğimiz, yeni bir parametre tanımladık. *Göreceli Agresiflik*, *Tedarik Gecikme Süresi*'nin *Stok Ayarlama Süresi*'ne bölümüne eşittir. Biz *Göreceli Agresiflik* için 4 değerini yeterince iyi ve uygulanabilir bir değer olarak öneriyoruz. Diğer bir deyişle *Stok Ayarlama Süresi*, *Tedarik Gecikme Süresi*'nin dörtte biri olarak öneriyoruz. *Tedarik Hattı Ağırlığı*'nın değişik gecikme düzenleri ve *Göreceli Agresiflik* değerleri için optimum değerleri tablo ile sunulmuştur.

İdeal Tedarik Hattı, ortalama tedarik süresi ile istenen tedarik oranının çarpımına eşittir. Bu hesaplama tedarik hattının ideal değerinde iken istenen tedarik oranını üretmesini sağlar. Yanlış hesaplanan bir *İdeal Tedarik Hattı* değeri sabit hal hatasına (stoğun hedefine ulaşamamasına) neden olur. Bu yüzden *İdeal Tedarik Hattı*'nın doğru hesaplanması önemlidir. Tek tedarikçili sistemlerde, istenen tedarik oranı, ortalama kayıp akışına eşittir. Bununla birlikte, istenen tedarik oranını belirleme çok tedarikçili sistemlerde kolay değildir. Bu tezde, *İdeal Tedarik Hattı* değerinin hesaplanması için genel bir formül önerilmekte ve öncelikli tedarikçi tercihleri ile tedarikçilerin üretim/sevkiyat kapasiteleri göze alınmaktadır.

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LIST OF SYMBOLS/ABBREVIATIONS

A	Coefficient matrix
I	Identity matrix
λ	Eigenvalue
α_s	1/Stock adjustment time
α_{SL}	1/Supply line adjustment time in Sterman, 1989a
β	Weight of supply line in Sterman, 1989a
ADT	Acquisition delay time
AF	Acquisition flow (acquisition flow from supply line to primary stock)
AF_i	Acquisition flow *i* (acquisition flow from *i* th stock of supply line to *i+1* st stock of supply line)
CF	Control flow
DS	Desired stock
DSL	Desired supply line
DT	Time step
LF	Loss flow
RA	Relative aggressiveness
S	Stock
SA	Stock adjustment
SAT	Stock adjustment time
SL	Supply Line
SL_i	*i* th stock of supply line
SLA	Supply line adjustment (supply line adjustment term in desired control flow)
TP	Total Penalty
WSL	Weight of supply line

1. INTRODUCTION

Human life is intertwined with complex social and economic systems. The dynamic nature of these systems requires a continuous decision making effort. However, many experiments show that we are not successful at dynamic decision making and, thus, managing dynamic systems (Brehmer, 1992; Diehl and Sterman, 1995; Moxness, 2000; Sterman, 1989, 2000; Sweeney and Sterman, 2000; Yasarcan, 2010). Hence, a considerable improvement in the process of decision making is necessary in order to obtain better outcomes from these systems.

The difficulty in controlling a dynamic system arises mainly from the internal complexity of that system. In addition to being poor decision makers in the presence of "dynamic complexity", we even fail at understanding the effects of the isolated individual complexity elements such as accumulation processes, feedback loops, delays, and nonlinearities (Diehl and Sterman, 1995; Hamilton, 1980; Moxness, 2000; Sterman, 1989a, 1989b; Sweeney and Sterman, 2000; Yasarcan, 2011).

Stock management is a widely encountered task in complex dynamic systems. This task can be found in physical, biological, managerial, or any other kind of system. In a stock management task, the aim of a human decision making process is to alter the system state towards or maintain it at a desired point (Diehl and Sterman, 1995; Sterman, 1987a, 1989, Chapter 17 in 2000; Yasarcan and Barlas, 2005; Yasarcan, 2010; Yasarcan, 2011). Alternatively, the desired system state can be a range instead of a single point (Barlas and Dalkiran, 2008; Herdem and Yasarcan, 2010; Sterman and Sweeney, 2002). As an internally rich dynamic task, stock management introduces difficulties for human decision makers, which may result in unwanted problematic dynamic behaviors.

Oscillatory dynamics observed in inventory management problems is an example for unwanted behavior because associated inventory holding costs, backordering costs or costs related to loss of sales, and loss of goodwill increases as a result of undesired oscillations (Barlas and Ozevin, 2004; Sterman, 1987a and 1989; Yasarcan, 2010). A reasonable inventory behavior is a stable yet quick approach of inventory to its desired level

(Yasarcan and Barlas, 2005; Yasarcan, 2011). In a blood glucose regulation problem, a reasonable dynamics is to have the blood glucose concentration level between a desirable range so as to prevent hypoglycemia and hyperglycemia (Herdem and Yasarcan, 2010). In a fire fighting task, a reasonable behavior is to control the spread of fire, diminish it, and, finally, make it stop. In a fire fighting simulation game, most participants could not prevent the spread of fire in the forest and some even let their base station burn down generating an undesired result (Brehmer, 1992).

In his famous work, Sterman (1989a) suggests an anchor-and-adjust heuristic as a representation of the managerial decision making process for a stock management task. Anchor-and-adjust is a widely used heuristic in stock management studies. This heuristic has three terms: expected loss from the stock; stock adjustment (the discrepancy between the desired and actual stock divided by a time parameter); supply line adjustment (the discrepancy between the desired and actual supply line of unfilled orders divided by a time parameter) (Barlas and Ozevin, 2004; Diehl and Sterman, 1995; Sterman, 1987a, 1989a, 1989b, and Chapter 17 in 2000; Yasarcan, 2010 and 2011; Yasarcan and Barlas, 2005a and 2005b). In this study, we focused on two different problems in anchor-and-adjust heuristic and analyzed each of these problems in a separate part. One is the selection of the values of decision making parameters; the other is deciding on the target values for the supply lines of a stock.

It is known that the presence of a supply line delay may lead to undesired oscillatory stock behavior (Barlas and Ozevin, 2004; Sterman, 1987a and 1989a; Yasarcan, 2010; Yasarcan and Barlas, 2005a and 2005b). The existence of a delay between the decisions and their results leads to misperceptions of feedback; decision makers misperceive the results of their decisions (Sterman, 1989a). In his study, Sterman (1989a) carried out a dynamic decision making experiment using human participants. The participants were asked to manage an inventory distribution system consisting of four cascading stock management tasks. He successfully modeled participants' decisions using the anchor-and-adjust heuristic and estimated the parameters of the heuristic based on the decisions of the human participants. The averages of these parameter estimates are far from the optimal values suggested in that paper.

There are three time parameters for the three terms of the anchor-and-adjust heuristic; one for each. Expectation formation is out of the scope of this paper. Therefore, we ignore the time parameter used in the expected loss term. For expectation formation, see Sterman (1987b). The two other time parameters are *Stock Adjustment Time* ($1/\alpha_S$ in Sterman, 1989a) used in the stock adjustment term and *Supply Line Adjustment Time* ($1/\alpha_{SL}$ in Sterman, 1989a) used in the supply line adjustment term. In general, it can be said that the existence and stability of oscillations in stock dynamics is determined by the values assigned to these two time parameters for a given delay duration (*Acquisition Delay Time*; λ in Sterman, 1989a) and delay order. The delay duration in this paper is named as *Acquisition Delay Time* and it stands for the lag between the control decisions and their effects on the stock. A few examples of *Acquisition Delay Time* can be listed as: the supply lead time in an inventory control system; the time required to hire and train employees in managing the level of human resources of a company; the actuator delay and the controller response delay in an engineering control system; the duration of time to digest and assimilate food in controlling one's level of fullness.

Alternatively, a weight coefficient can be used in the supply line adjustment term so that a single adjustment time can be used instead of explicitly using two separate adjustment times. This coefficient reflects the relative importance given to the supply line compared to the stock. Therefore, this weight is called *Weight of Supply Line* (β in Sterman, 1989a) and it is equal to *Stock Adjustment Time* divided by *Supply Line Adjustment Time*. The supply line can fully be considered by setting *Weight of Supply Line* equal to 1, which corresponds to using the same adjustment time for stock adjustment and supply line adjustment terms. Fully considering supply line means that the decision maker gives the same importance to the discrepancies between the desired and actual levels of both the stock and its supply line. Giving the same importance to the stock and its supply line effectively reduces the stock management task to a first order system, which cannot oscillate. Hence, *Weight of Supply Line* equal to unity ensures non-oscillatory stock behavior regardless the delay duration and order (Barlas and Ozevin, 2004; Sterman, 1989a and Chapter 17 in 2000; Yasarcan and Barlas, 2005a and 2005b).

Stock Adjustment Time ($1/\alpha_S$ in Sterman, 1989a) and *Weight of Supply Line* (β in Sterman, 1989a) are two of the important decision parameters of the anchor-and-adjust heuristic. According to Sterman, human decision makers assign non-optimum values to these parameters, which results in unwanted stock dynamics. There are many other papers that also report problematic stock behaviors obtained by participants (see for example, Barlas and Ozevin, 2004; Yasarcan, 2010). As the stock management task is the most common dynamic decision making problem and as the human decision makers have problems managing the task, there is a strong need to have a rule of thumb in assigning values to the decision parameters *Stock Adjustment Time* and *Weight of Supply Line* for the different cases of the problem. As an answer to this need, we investigate a generic stock management task in continuous and discrete time with a material supply line delay of different delay durations and orders.

The existence and stability of oscillations in stock dynamics is a function of the order of the delay structure, *Acquisition Delay Time* (delay duration), *Weight of Supply Line*, and *Stock Adjustment Time*. In Chapter 5 of his PhD thesis, Yasarcan (2003) reported the critical values of the ratio between the two parameters *Stock Adjustment Time* and *Acquisition Delay Time*. Those critical values determine the changes in the dynamics of the stock from no-oscillations to stable oscillations and stable oscillations to unstable oscillations. Being inspired by Yasarcan's approach, we introduce a new parameter that we name as *Relative Aggressiveness* and define it to be equal to *Acquisition Delay Time* divided by *Stock Adjustment Time*. A low *Stock Adjustment Time* value implies aggressive corrections and a high value of the parameter implies smooth corrections; *Stock Adjustment Time* is a measure of aggressiveness in decision making. Hence, *Relative Aggressiveness* is a measure of aggressiveness in making corrections relative to *Acquisition Delay Time*. Based on the findings reported in Yasarcan (2003) and the unreported extensive simulation analysis we carried out as a part of this study, we infer that the nature of the stock behavior is determined by the two ratios for a given delay order: *Weight of Supply Line* and *Relative Aggressiveness*. Once a reasonable value for *Relative Aggressiveness* is obtained, a sound *Stock Adjustment Time* value can be calculated for any given value of *Acquisition Delay Time*. Thus, introducing *Relative Aggressiveness* puts the selection of *Stock Adjustment Time* into an analytical framework. Note that, Yasarcan (2003) obtained the critical values assuming *Weight of Supply Line* equal to zero and for the delay orders 0, 1, 2, and ∞. In

Chapter 3 of this thesis, we carried out simulation runs for the different values of *Weight of Supply Line*, for the different values of the other parameters, and for the delay orders 0, 1, 2, 3, 4, 8, and ∞.

In Chapter 4 of this thesis, we studied multi-supplier stock management systems. The sourcing success of a manufacturer does not only depend on the ordering strategies, but it also depends on supplier selection. A firm should use multiple-sourcing strategy instead of a single-sourcing strategy in order to reduce procurement risk (Arda and Hennet, 2006; Chiang, 2001; Chiang and Benton, 1994; Jokar and Sajadieh, 2008; Minner, 2003; Ramasesh, 1991; Sculli and Shum, 1990; Sculli and Wu, 1981; Thomas and Tyworth, 2006). In the presence of stochastic lead times, multiple-sourcing strategy reduces the effective lead time (Minner, 2003). Multiple sourcing also reduces the dependency on a single supplier, thus the power of supplier over the buyer (Burke, Carrillo and Vakharia, 2007; Newman, 1989). Some firms give scores to its suppliers, and decide on their priority levels. They give orders according to the priority levels of its suppliers (Burke, Carrillo and Vakharia, 2007).

Determining the target value for a supply line, which is called *Desired Supply Line*, is important when anchor-and-adjust heuristic is used. *Desired Supply Line* is calculated by multiplying expected acquisition lag and desired acquisition rate (Sterman, 2000). In a single-supplier stock management problem, desired acquisition rate for a stock is equal to the expected loss flow from that stock. Therefore, *Desired Supply Line* is equal to expected acquisition lag times expected loss flow. However, determining desired acquisition rate, thus, calculating *Desired Supply Line*, is not that straightforward in the presence of multiple suppliers.

A stock having multiple supply lines can be seen in inventory management, human resource management, capacity management, and personnel training. For example, firms have different human resources management processes. Some firms use an internal human resources department. Some outsource their human resources needs to private agents. Additionally, some of them use both an internal department and private agents. Each entity who deals with human resources management has a different operational mechanism which has its own working capacity and hiring lead time. The hiring/firing process of each entity corresponds to a different supply line for the human workforce of a firm. Firms using multi-sourcing strategies need to decide on the utilization level of each supplier. They should also determine *Desired Supply Line* values for each of those supply lines, which is the main issue examined in Chapter 4.

2. METHODOLOGY AND BACKGROUND INFORMATION

In this thesis, System Dynamics (SD) methodology is used for modeling and simulation. Causal-loop diagrams and stock-flow diagrams of SD are developed as a part of the modeling process. Stocks and flows are the main building blocks used in stock-flow diagrams and they are used to model accumulation processes, which exist in every system with some degree of internal dynamics.

An accumulation process consists of a stock and the flows attached to this stock. Stocks are the state variables and they give memory to the system. Flows are the "rate of change" of stocks; they are the sources of change in a system. Stocks accumulate flows and flows characterize the change of stocks over time (Barlas, 2002; Forrester, 1961, 1971; Sterman, 2000). Since a stock is an accumulation, it has inertia and can only be changed gradually in time via the flow or flows attached to it. Note that, accumulation processes are calculated by summation in discrete time models and integration in continuous time models. According to the literature, humans have problems at understanding the effects of an accumulation process, whether it is discrete or continuous, even if the other elements of dynamic complexity (i.e. feedback loops, delays, and nonlinearities) are not present in the task (Cronin, Gonzalez, and Sterman, 2009; Sterman, 1989, 2002, 2008; Sweeney and Sterman, 2000).

A causal-loop diagram consists of one or more feedback loops. A feedback loop is a chain of cause-effect relations such that a change in any of its variables affects all other variables on this chain successively and, ultimately, re-affects itself (Barlas, 2002; Forrester, 1961, 1971; Sterman, 2000). "... a feedback loop is a succession of cause-effect relations that start and end with the same variable. It constitutes a circular causality, only meaningful dynamically, over time." (Barlas, 2002). On a feedback loop, no distinction exists between independent and dependent variables; they are all interdependent. Similar to accumulation processes, feedback loops also contribute to the dynamic complexity of systems. The existence of a feedback loop in a dynamic problem is a must to have endogenously created rich dynamics. Therefore, a feedback loop is a potential source of a counterintuitive behavior. Note that it is possible to model an isolated accumulation process. However, modeling an isolated feedback loop is not possible; every feedback loop contains at least one stock and one flow (Barlas, 2002; Forrester, 1961, 1971; Sterman, 2000). The problems caused by poor understanding of feedback loops are mainly studied under "misperceptions of feedback" phenomenon (Brehmer, 1992; Diehl and Sterman, 1995; Moxness, 2000; Sterman, 1987, 1989, 2000).

In the generic stock management model, there exists a stock accumulation process, a delay structure, and two decision making feedback loops. Similar to accumulation processes and feedback loops, existence of a delay causing structure also contributes to the dynamic complexity. Hence, it is one of the causes of the misperceptions of feedback in a stock management task.

A delay is the existence of a lag between an input and its resultant output. In modeling, delay is simply formulated such that the output of a delay formulation seeks the input of that formulation. Representation of a delay requires storing past values of the input variable. Therefore, delay formulations naturally involve stocks and flows. In order to prevent conceptual confusion, in this study, the stocks and flows within a delay formulation will not be considered as separate accumulation processes, but the totality of that delay structure will be considered as another type of dynamic complexity element. A delay structure is characterized by its average duration and its order. Order of a delay structure is equal to the number of stocks in that structure and it defines how the output seeks the input to that structure (i.e. the shape of the dynamic behavior of the output). See the output behaviors for delay orders 1, 3, 8 and infinity from Figure 2.1 (delay duration is taken as 5).

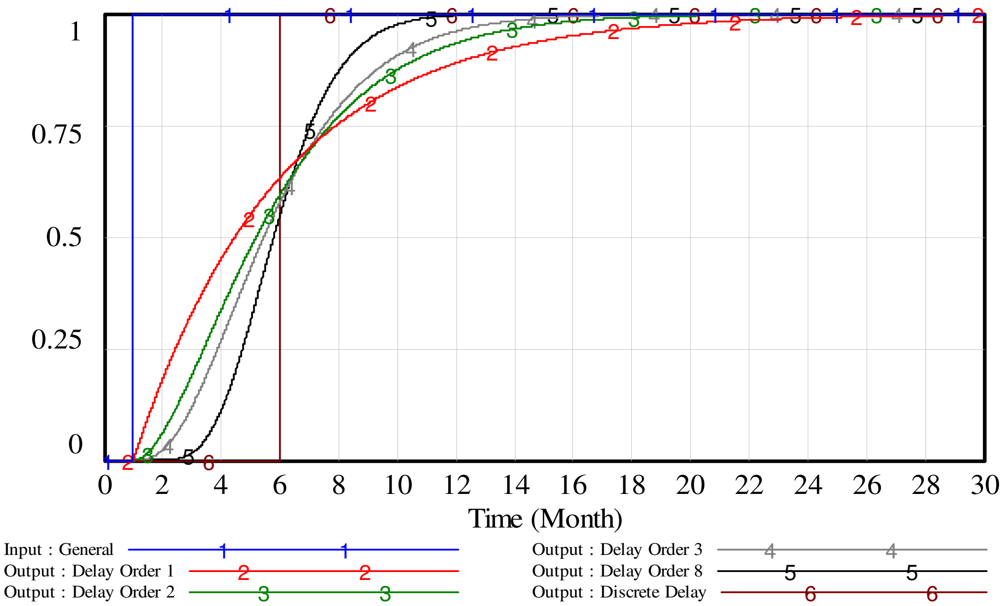

Figure 2.1. Output Behavior for Different Delay Orders

It is possible to conceptualize a structure without a delay. However, there exist no real dynamic systems without delays. For example, training a worker, delivering an order, deciding on how many workers to hire or fire, processing customer orders, creating inventory reports, forecasting sales; they all take some time.

3. A RULE OF THUMB FOR THE SELECTION OF DECISION MAKING PARAMATER VALUES

Anchor-and-adjust heuristic is a representation of managerial decision making process for the stock management task. The decision parameters of the anchor-and-adjust heuristic are *Stock Adjustment Time* (the average time to close the discrepancy between the actual and desired stock) and *Weight of Supply Line* (the importance assigned to the supply line relative to the stock). Selection of these decision parameter values determines the success of the heuristic and, thus, managerial decisions. Causal loop diagram of the generic stock management task controlled by the anchor-and-adjust heuristic can be seen in Figure 3.1. The negative feedback-loop consisting of *Supply Line* and *Acquisition Flow* describes the decay process that empties *Supply Line* and fills-in *Stock*. The anchor-and-adjust heuristic that we used has one anchor, which is *Loss Flow*, and two adjustment terms: *Stock Adjustment* and *Supply Line Adjustment*. Each adjustment term introduces a decision making feedback loop. The aim of the negative feedback-loop containing *Stock*, *Stock Adjustment*, *Control Flow*, *Supply Line*, and *Acquisition Flow* is to close the gap between *Stock* and its desired level, *Desired Stock*. The aim of the negative feedback-loop containing *Supply Line*, *Supply Line Adjustment*, and *Control Flow* is to close the gap between *Supply Line* and its desired level, *Desired Supply Line*.

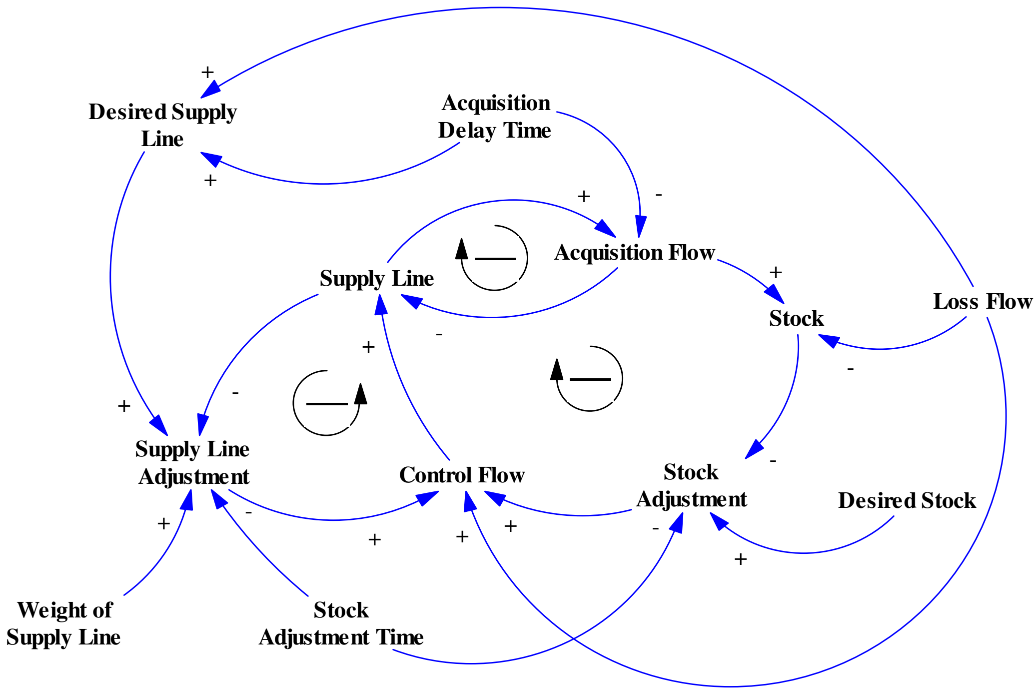

Figure 3.1. Causal Loop Diagram of the Stock Management Task

3.1. Relative Aggressiveness

The main goal of this work is to improve managerial decision making by obtaining a rule thumb for determining *Stock Adjustment Time* and *Weight of Supply Line* values. The other factors (such as *Loss Flow*, *Acquisition Delay Time*, and the order of the delay structure) describe the physical aspects of the stock management structure and we assumed that they cannot be decided by the decision maker. In the stock management task controlled by the anchor-and-adjust heuristic, the existence and stability of oscillations observed in stock dynamics is a function of the order of the delay structure, *Acquisition Delay Time* (delay duration), *Weight of Supply Line*, and *Stock Adjustment Time*. *Stock Adjustment Time* and *Weight of Supply Line* are decision making parameters and *Acquisition Delay Time* and the order of the delay structure describe the physical characteristics of the lead time. To attain a reasonable stock behavior in the stock management task, the values of *Stock Adjustment Time* and *Weight of Supply Line* should be determined by considering the structure and duration of the lead time.

Although the nominal values of *Acquisition Delay Time* and *Stock Adjustment Time* affect the stock behavior, it is their ratio (together with the order of the delay structure and *Weight of Supply Line*) that determines the existence and stability of oscillations. For example, if the stock is showing damping oscillations for the given set of parameter values, changing the values of *Acquisition Delay Time* and *Stock Adjustment Time* by the same ratio will not make the stock stop oscillating or show unstable oscillations, but the period and amplitude of the oscillations will be affected (see Appendix A).

Therefore, we introduce a new decision parameter to the model called *Relative Aggressiveness* which is *Acquisition Delay Time* divided by *Stock Adjustment Time*. Stock dynamics is determined by the two ratios: *Weight of Supply Line* and *Relative Aggressiveness*. Once a good value is determined for *Relative Aggressiveness*, a *Stock Adjustment Time* value can be calculated by using Equation 3.17. Therefore, we carried out many simulation runs for all values of *Weight of Supply Line* and *Relative Aggressiveness* (within a range and at a selected precision level) and for 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, and 8^{th} order and discrete (infinite order) delay structures in continuous and discrete time.

3.2. Description of the Stock Management Structure

In this section, we present the stock management structure used in this study. The stock-flow diagram given in Figure 3.2, and the corresponding equations belong to the stock management structure with a second order material supply line delay. For information about stock management structure, see Sterman (1987a, 1989a, and chapter 17 in 2000) and Yasarcan and Barlas (2005a). In order not to use unnecessary space, the stock-flow diagrams and their corresponding equations for other delay orders are not presented. For information about the order of delay structures, see Barlas (2002) and chapter 11 in Sterman (2000).

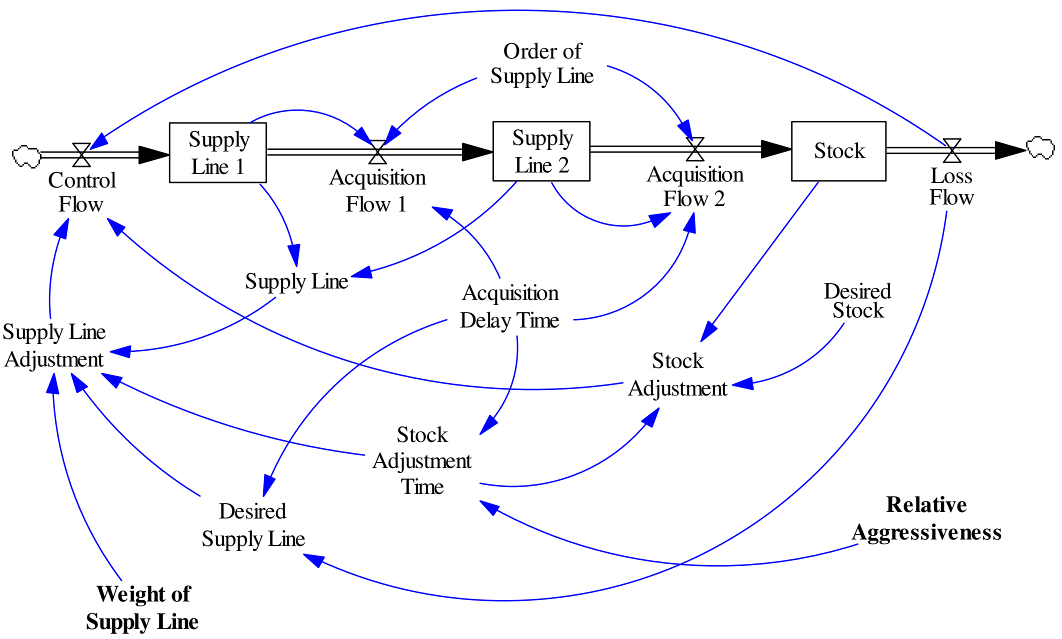

Figure 3.2. Stock-Flow Diagram of the Stock Management Task with a 2nd Order Supply Line

Stock is the main level of our model. We try to maintain *Stock* at a desired point. The penalties, which are used for our analysis, are defined with respect to the level of *Stock* and its desired level. In this study, we use Euler integration method to simulate, which is reflected in the stock equations which are given below:

$$Stock_0 = Desired\ Stock\ Level \quad [item] \tag{3.1}$$

$$Stock_{t+DT} = Stock_t + (Acquisition\ Flow\ 2 - Loss\ Flow) \times DT \quad [item] \tag{3.2}$$

The number of supply line stocks is determined by the order of supply line.

$$Supply\ Line\ 1_0 = \frac{Desired\ Supply\ Line}{Order\ of\ Supply\ Line} \quad [item] \tag{3.3}$$

$$Supply\ Line\ 1_{t+DT} = Supply\ Line\ 1_t + \begin{pmatrix} Control\ Flow \\ - Acquisition\ Flow\ 1 \end{pmatrix} \times DT \quad [item] \tag{3.4}$$

$$Supply\,Line\,2_0 = \frac{Desired\ Supply\ Line}{Order\ of\ Supply\ Line} \quad [item] \tag{3.5}$$

$$Supply\,Line\,2_{t+DT} = Supply\,Line\,2_t + \begin{pmatrix} Acquisition\,Flow\,1 \\ -Acquisition\,Flow\,2 \end{pmatrix} \times DT \quad [item] \tag{3.6}$$

Model is initiated at its equilibrium point; state variables (stocks) are initiated at their equilibrium levels (equations 3.1, 3.3, and 3.5). A shock is given to the model by increasing *Desired Stock* by 1 unit at time 1 (Equation 3.12).

Flow equations are as follows:

$$Loss\ Flow = 2 \quad [item\ /\ time] \tag{3.7}$$

$$Control\ Flow = \begin{pmatrix} Loss\,Flow + Stock\,Adjustment \\ + Supply\,Line\,Adjustment \end{pmatrix} \quad [item/time] \tag{3.8}$$

$$Acquisition\,Flow\,1 = \frac{Supply\ Line\ 1}{\begin{pmatrix} Acquisition \\ Delay\,Time \end{pmatrix} \Big/ \begin{pmatrix} Order\ of \\ Supply\ Line \end{pmatrix}} \quad [item/time] \tag{3.9}$$

$$Acquisition\,Flow\,2 = \frac{Supply\ Line\ 2}{\begin{pmatrix} Acquisition \\ Delay\,Time \end{pmatrix} \Big/ \begin{pmatrix} Order\ of \\ Supply\ Line \end{pmatrix}} \quad [item/time] \tag{3.10}$$

As mentioned in the previous section, expectation formation is out of the scope of this work. Therefore, *Loss Flow* is taken as constant (2 items per time) and assumed to be known by the decision maker. This assumption is reflected in Equation 3.7. *Control Flow* is the decision reflecting the instantaneous control decision. It is the input of our material supply line delay structure. The number of acquisition flows is determined by the order of supply line delay like the number of supply line stocks. The last acquisition flow is the output of our material supply line delay structure. It is the instantaneous result of our control decisions.

Model constants and the other model equations are:

$$Acquisition\,Delay\,Time = 8 \quad [time] \tag{3.11}$$

$$Desired\ Stock = \begin{Bmatrix} 9, & Time < 1 \\ 10, & Time \geq 1 \end{Bmatrix} \quad [item] \tag{3.12}$$

$$Desired\ Supply\,Line = Acquisition\ Delay\ Time \times Loss\ Flow \quad [item] \tag{3.13}$$

$$Supply\,Line = Supply\ Line\ 1 + Supply\ Line\ 2 \quad [item] \tag{3.14}$$

$$\begin{pmatrix} Supply \\ Line \\ Adjustment \end{pmatrix} = \frac{\begin{pmatrix} Weight\ of \\ Supply\ Line \end{pmatrix} \times \begin{pmatrix} Desired\ Supply\,Line \\ -\ Supply\,Line \end{pmatrix}}{Stock\ Adjustment\,Time} \quad [item/time] \tag{3.15}$$

$$Stock\ Adjustment = \frac{Desired\ Stock\ Level - Stock}{Stock\ Adjustment\ Time} \quad [item/time] \tag{3.16}$$

$$Stock\ Adjustment\ Time = \frac{Acquisition\ Delay\ Time}{Relative\ Aggressiveness} \quad [item/time] \tag{3.17}$$

The values of *Weight of Supply Line* and *Relative Aggressiveness* are not presented with other equations because they are the experimental parameters; the values of these parameters correspond to different ordering policies. Note that both *Weight of Supply Line* and *Relative Aggressiveness* are dimensionless parameters. In addition to the model equations, the penalty equations 3.18 and 3.19 are introduced so as to compare the performances of the different ordering policies.

$$Total\ Penalty_0 = 0 \quad [item \cdot time] \tag{3.18}$$

$$Total\ Penalty_{t+DT} = Total\ Penalty_t + \begin{vmatrix} Desired\ Stock\ Level \\ -\ Stock \end{vmatrix} \times DT \tag{3.19}$$

3.3. Design for Simulation Experiments

As we have already mentioned, the aim is to find a good set of values for *Weight of Supply Line* and *Relative Aggressiveness*. After an extensive pilot study, we selected the range of *Weight of Supply Line* as [0.0, 1.6] and the range of *Relative Aggressiveness* as [0.1, 6.0]. The range for *Weight of Supply Line* is divided into 65 equal distance points; the gap between two successive points is 0.025. The range for *Relative Aggressiveness* is divided into 60 equal distance points; the gap between two successive points is 0.1. Therefore, the total number for simulations is 3,900 for each and every one of the delay orders 1, 2, 3, 4, 8, and infinite.

The rest of the model parameters and initial values presented in the previous section are arbitrarily selected, but they are kept constant for all simulation runs in order to have consistency. Any change in those parameter and initial values generates a numerically different *Total Penalty* calculated by the equations 3.18 and 3.19. However, the change has absolutely no effect on the relative values of the two *Total Penalty* values generated by using two different pair of values for the parameter set (*Weight of Supply Line*, *Relative Aggressiveness*).

The difficulty faced in the pilot studies of this experiment was the numerical precision and errors introduced by the limitations of the simulation software. In our simulations, we use the Euler integration method and set *DT* (simulation time step) equal to $1/32$ so as to keep these numerical concerns at an acceptable level. Also, the selection of simulation length is important for a fair comparison of different simulation runs. The settling down of the stock behavior and keeping the accumulated numerical errors at an acceptable level are the main concerns of the simulation length selection process. Taking these into consideration, the simulation length is selected as 250 as a part of our standard simulation setting. The selected *DT* and final time values give a total of 8,000 simulation steps for each simulation run.

For the sake of completeness, the stock management task with a discrete delay structure is examined under similar settings in discrete time too.

3.4. The Effects of Weight of Supply Line and Relative Aggressiveness on the Dynamics of the Stock

In figure 3.3, we assigned 0 in the first run and 0.5 in the second run to *Weight of Supply Line*. In the first run, *Weight of Supply Line* is badly chosen and this leads to an unstable stock behavior. In the second run, *Weight of Supply Line* is chosen well and this produces a stable stock behavior.

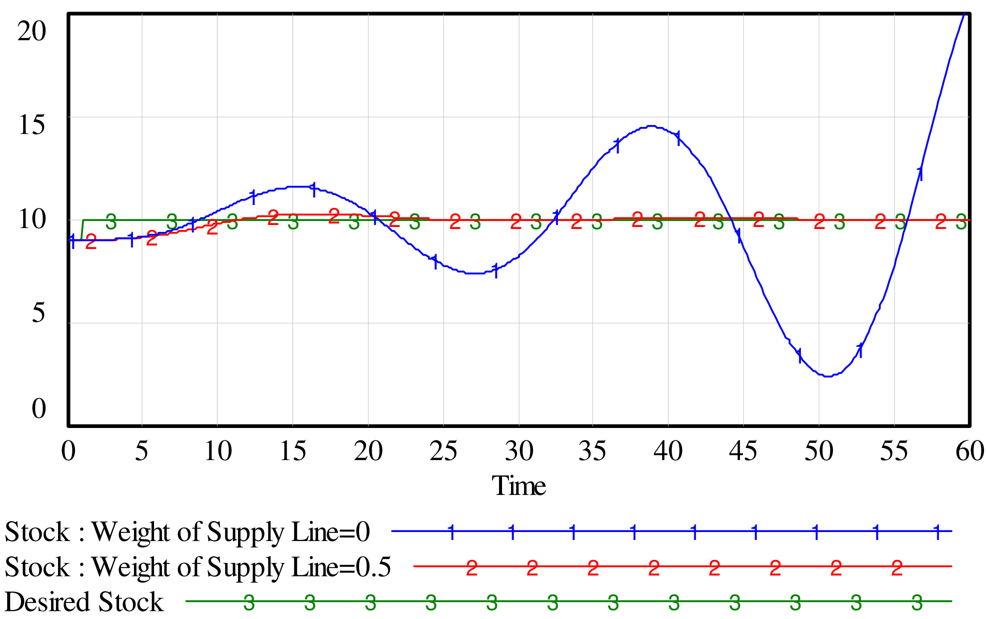

Figure 3.3. Stock Behavior with *Weight of Supply Line* Values 0 and 0.5 when Delay Order is 3 and *Relative Aggressiveness* is 5

In figure 3.4, *Relative Aggressiveness* equal to 1 is used in the first run and *Relative Aggressiveness* equal to 20 is used in the second run. The behavior observed in the first run is reasonable. However, the behavior observed in the second run is problematic because unstable oscillation is costly and unwanted. Hence, the selection of the decision parameter is critical in obtaining a desirable stock behavior.

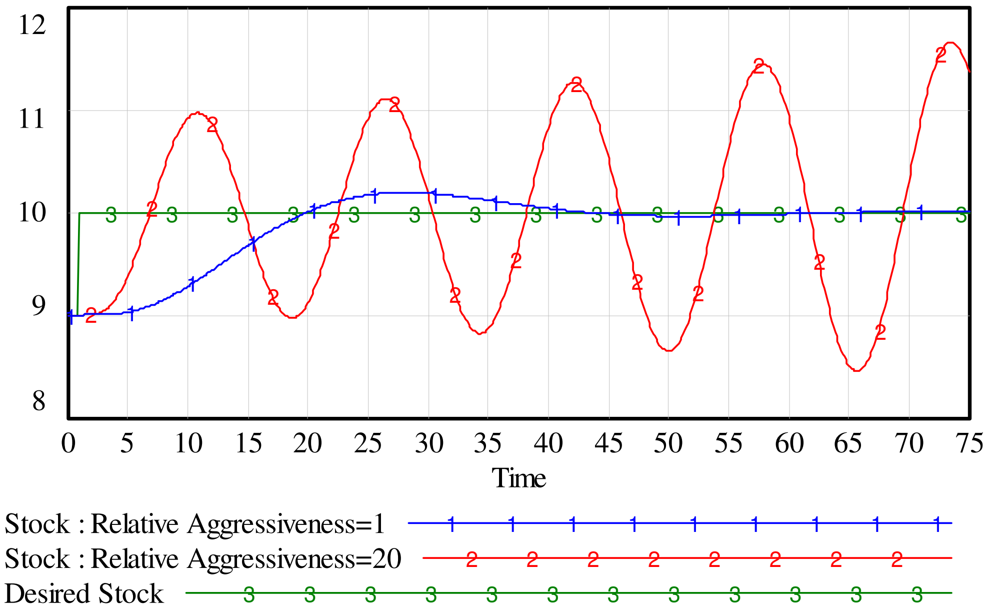

Figure 3.4. Stock Behavior with *Relative Aggressiveness* Values 1 and 20 when Delay Order is 3 and *Weight of Supply Line* is 0.2

Note that, increasing *Relative Aggressiveness* has a destabilizing effect on stock behavior. However, when *Weight of Supply Line* is chosen accordingly, possible undesired stock behavior can be prevented.

3.5. Contour Plots of Total Penalty

Contour plots help us to examine three variables in two dimensions. The contour plots in figures 3.5 and 3.6 show the relationship between *Weight of Supply Line*, *Relative Aggressiveness*, and *Total Penalty*. *Total Penalty* is represented as the contour level in the plots.

Figure 3.5 gives contour plots of the generated *Total Penalty* values for all simulation runs in continuous time; there is one plot for each and every one of the delay orders 1, 2, 3, 4, 8, and infinite. In Figure 3.6, there is one contour plot for the simulation runs in discrete time. That plot is obtained by using a discrete (infinite order) delay structure in the stock management task. The darker areas in the contour plots represent lower *Total Penalty* values and the brighter areas represent higher values. From all contour plots (figures 3.5 and 3.6), it can be observed that the effects of *Weight of Supply Line* and *Relative Aggressiveness* on *Total Penalty* are not independent from each other. In general, both low and high values of *Weight of Supply Line* generate high penalties as it can be seen from the left and the right sides of the contour plots. Low *Relative Aggressiveness* values also produce high penalties; the bottom side of the contour plots is bright white. A setting with a high *Relative Aggressiveness* and a low *Weight of Supply Line* generates a high penalty (see the upper left of the contour plots especially for delay orders higher than one), but a setting with a high *Relative Aggressiveness* and a well-selected *Weight of Supply Line* generates a low penalty (see the dark areas in the upper side).

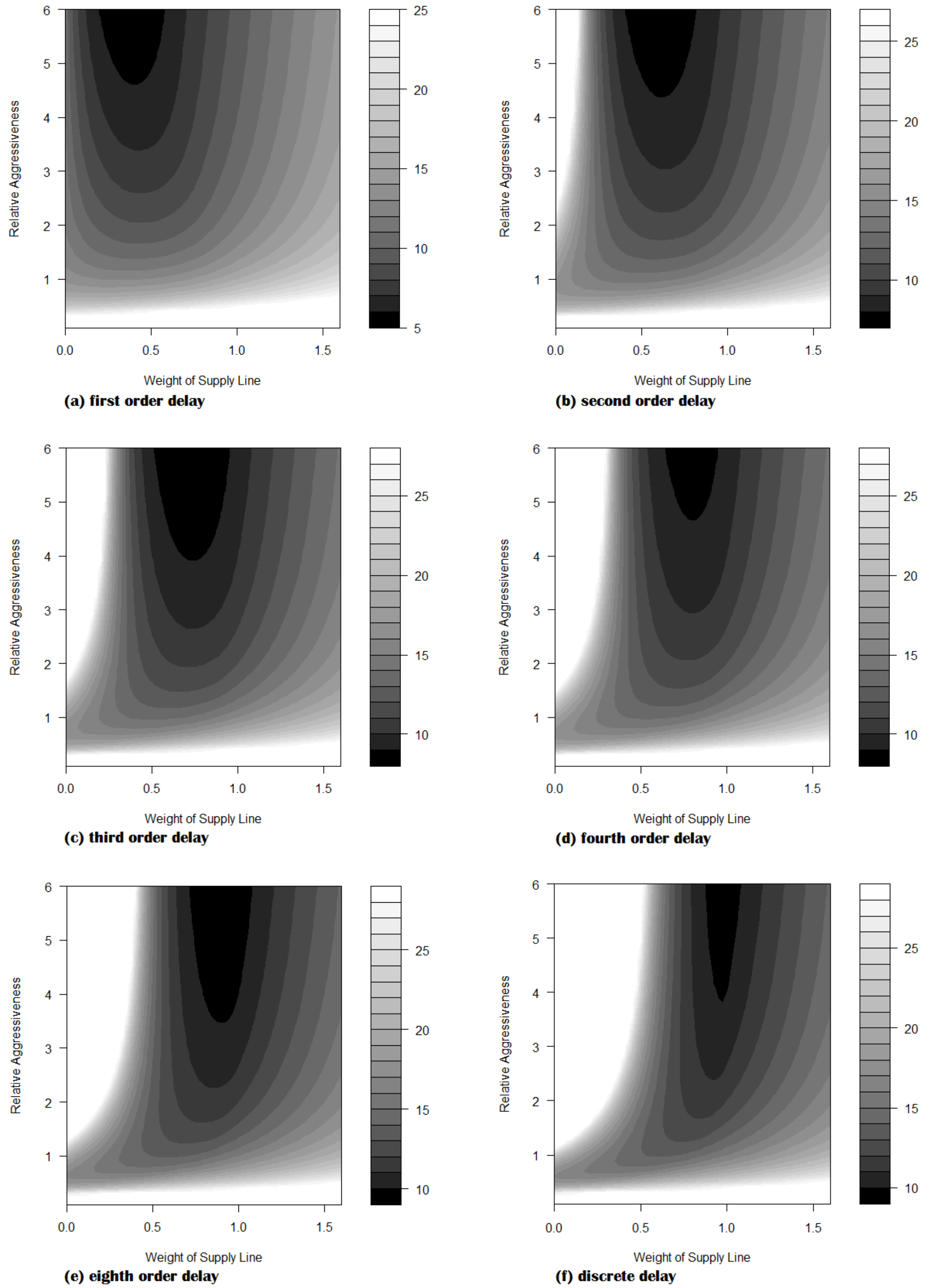

Figure 3.5. Contour Plots of *Total Penalty* Values in Continuous Time

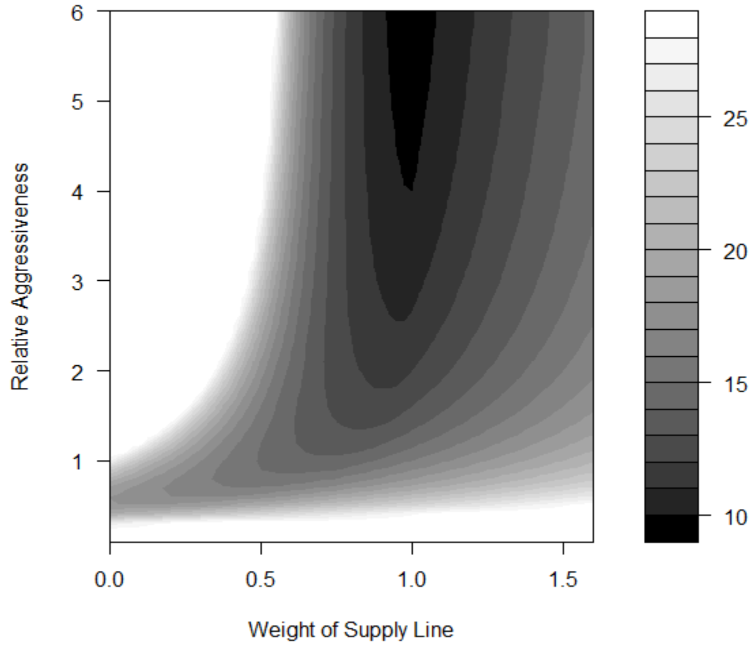

Figure 3.6. Contour Plot of *Total Penalty* Values in Discrete Time (Discrete Delay)

Figures 3.5 and 3.6 show that, increasing *Relative Aggressiveness* decreases the generated *Total Penalty* values when *Weight of Supply Line* is adjusted accordingly (some optimum values of *Weight of Supply Line* is presented in Table 3.1). However, the decrease in *Total Penalty* values becomes insignificant for big *Relative Aggressiveness* values. Even though there is no theoretical upper limit for *Relative Aggressiveness*, there are practical limits in real life. A high *Relative Aggressiveness* value implies a low *Stock Adjustment Time* value. In practice, a very low *Stock Adjustment Time* value may create problematic behavior due to factors that are not accounted for in our model (Yasarcan and Barlas, 2005b). Also, increasing Relative Aggressiveness has diminishing returns. Therefore, we suggest 4 as a desirable value for *Relative Aggressiveness*. It does not create problematic behavior in real systems and it generates reasonable *Total Penalty* values.

3.6. Optimum Weight of Supply Line Values for Different Delay Orders and Relative Aggressiveness Values

Table 3.1 presents the optimum *Weight of Supply Line* values for delay orders 1, 2, 3, 4, 8, and infinite, and for *Relative Aggressiveness* 1, 2, 3, 4, 5, and 6. The optimum values for *Relative Aggressiveness* = 4 is emphasized by bold fonts.

The optimum value for *Weight of Supply Line* is between zero and unity for any parameter setting and delay structure of the stock management task presented in this paper. For a well-selected *Relative Aggressiveness* value, the optimum shifts towards unity as delay order increases (see figures 3.5 and 3.7 and Table 3.1). The highest optimum values for *Weight of Supply Line* are obtained from the stock management task with a discrete delay structure. When the simulations runs are taken in discrete time instead of continuous time, the optimum further shifts towards one (see figures 3.5, 3.6, and 3.7 and Table 3.1).

Table 3.1. Optimum *Weight of Supply Line* Values for the Corresponding Delay Orders and *Relative Aggressiveness* Values

		Continuous Time						**Discrete Time**
		Delay Order 1	**Delay Order 2**	**Delay Order 3**	**Delay Order 4**	**Delay Order 8**	**Discrete Delay**	**Discrete Delay**
Relative Aggressiveness	1	0.325	0.475	0.525	0.550	0.600	0.625	0.700
	2	0.450	0.625	0.725	0.775	0.850	0.900	0.925
	3	0.425	0.625	0.750	0.800	0.900	0.950	0.975
	4	**0.425**	**0.625**	**0.750**	**0.800**	**0.900**	**0.975**	**1.000**
	5	0.400	0.600	0.725	0.800	0.925	0.975	1.000
	6	0.375	0.575	0.725	0.800	0.925	1.000	1.000

3.7. The Effect Weight of Supply Line on Total Penalty

In Figure 3.7, *Total Penalty* is plotted against *Weight of Supply Line* for delay orders 1, 2, 3, 4, 8, and infinite when *Relative Aggressiveness* is taken as 4. The increase in delay order increases *Total Penalty* values. In order to handle the control problems caused by high delay orders, *Weight of Supply Line* should be increased. However, it is seen from figure 3.7 that increasing *Weight of Supply Line* beyond 1 does not reduce penalties. On the contrary, increasing *Weight of Supply Line* beyond 1 increases the penalty values. It is also observed that when *Weight of Supply Line* is above 1, all delay structures produce the same exact penalty values regardless of their orders.

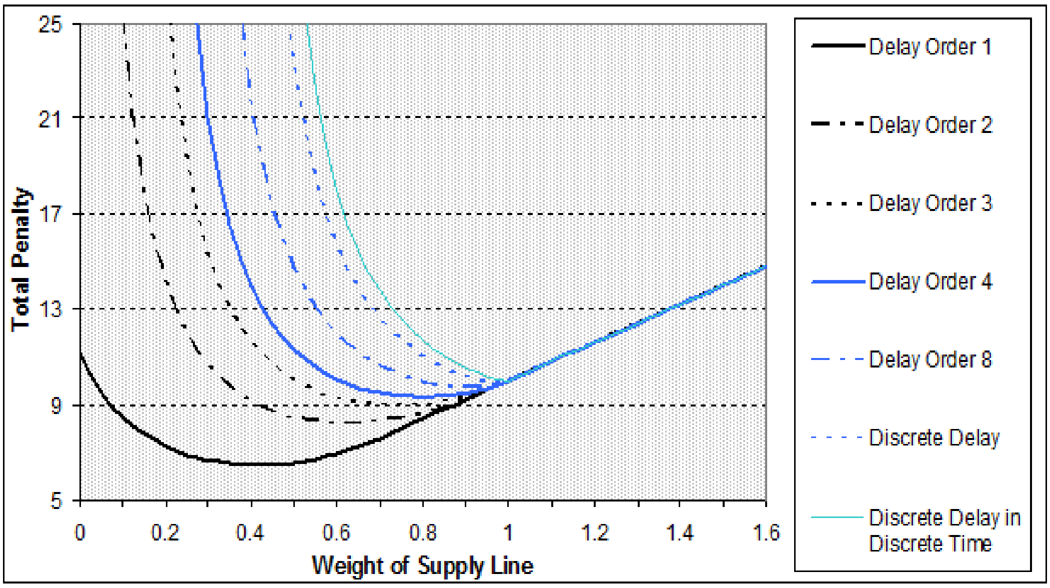

Figure 3.7. *Total Penalty* plotted against *Weight of Supply Line* when *Relative Aggressiveness* = 4

3.8. The Effect Acquisition Delay Time on Total Penalty

The effect of *Acquisition Delay Time* on *Total Penalty* can be observed from figures 3.8 and 3.9. Delay order is taken as 1 in Figure 3.8 and infinite in Figure 3.9. Four different sets are presented in both figures; the sets have the same model parameters except *Relative Aggressiveness* values. The *Relative Aggressiveness* values of 0.5, 1, 4, and 32 are used. The sets are chosen in order not to produce an unstable stock behavior, because a fair penalty comparison cannot be obtained in an unstable system. It is seen that *Acquisition Delay Time* has a linear effect on penalties. We also observe that increasing *Relative Aggressiveness* after a certain level does not significantly reduce the penalties, which is consistent with our previous observations. Previously, this level was decided as 4 from figures 3.5 and 3.6.

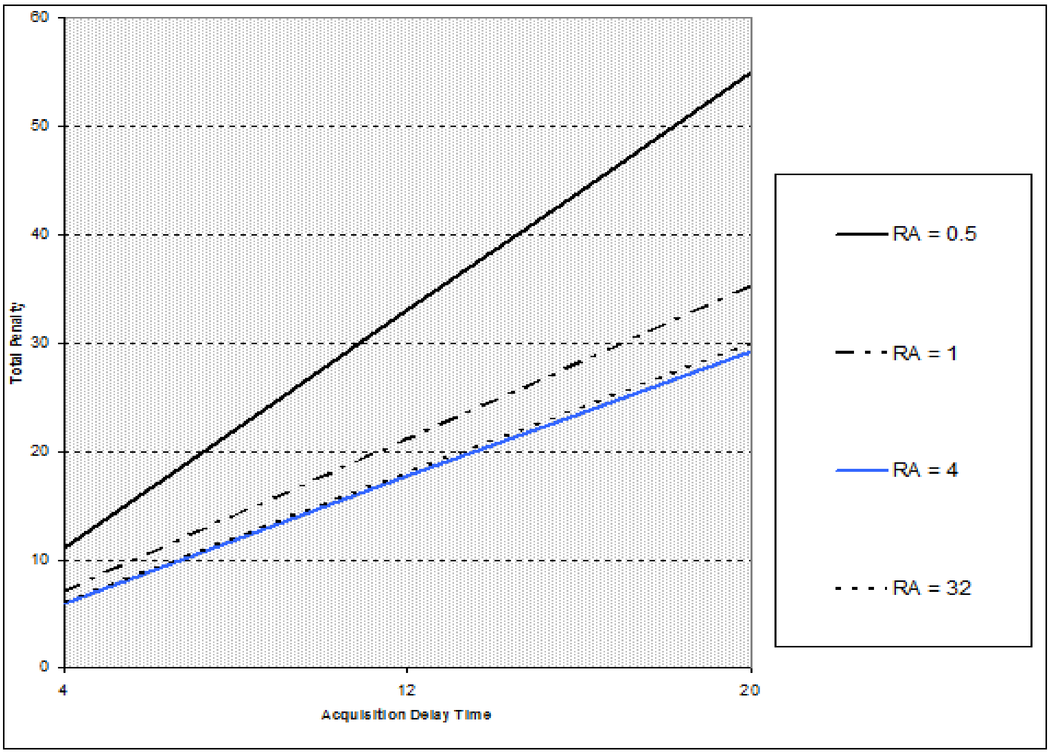

Figure 3.8. *Acquisition Delay Time* vs. *Total Penalty* when *Weight of Supply Line* = 0.75 and Delay Order is 1

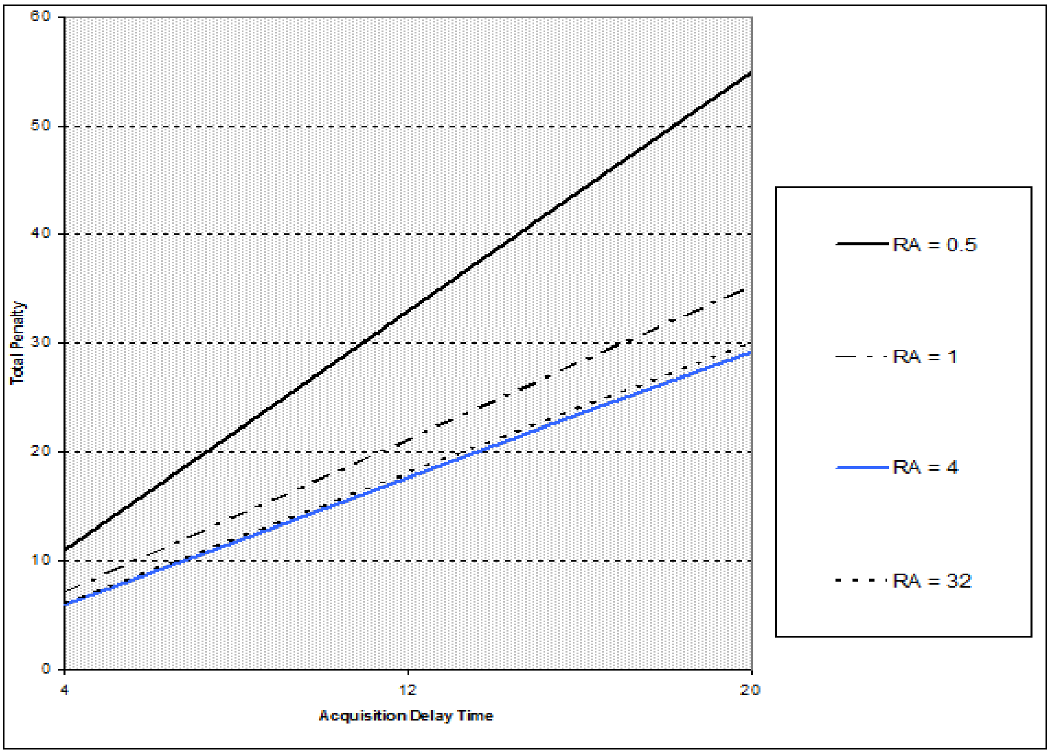

Figure 3.9. *Acquisition Delay Time* vs. *Total Penalty* when *Weight of Supply Line* = 0.75 and Delay is Discrete (Continuous Time)

4. DESIRED SUPPLY LINE VALUE DETERMINATION FOR MULTI-SUPPLIER SYSTEMS

In a stock management task, the goal is maintaining a stock at a desired level. This is achieved by adjusting for the supply line and the corresponding stock at the same time. To adjust for the supply line, its desired level should be chosen appropriately. We developed formulations for *Desired Supply Line* value calculation for multi-supplier systems in the presence of constant *Loss Flow* and stochastic *Loss Flow* with a known stationary mean.

4.1. Generic Formulations of Desired Supply Line

The general formula for *Desired Supply Line* is seen in Equation 4.1:

$$Desired\ Supply\ Line = Acquisition\ Delay\ Time \times Loss\ Flow \tag{4.1}$$

Equation 4.1 is valid for a stock with a single supply line. It needs to be adjusted when a stock has multiple suppliers and, thus, multiple supply lines. When there are *n* supply lines attached to a stock, each supply line needs to have its own *Desired Supply Line* value. As an example, a stock management system having 2 supply lines attached to a stock is presented in Figure 4.1.

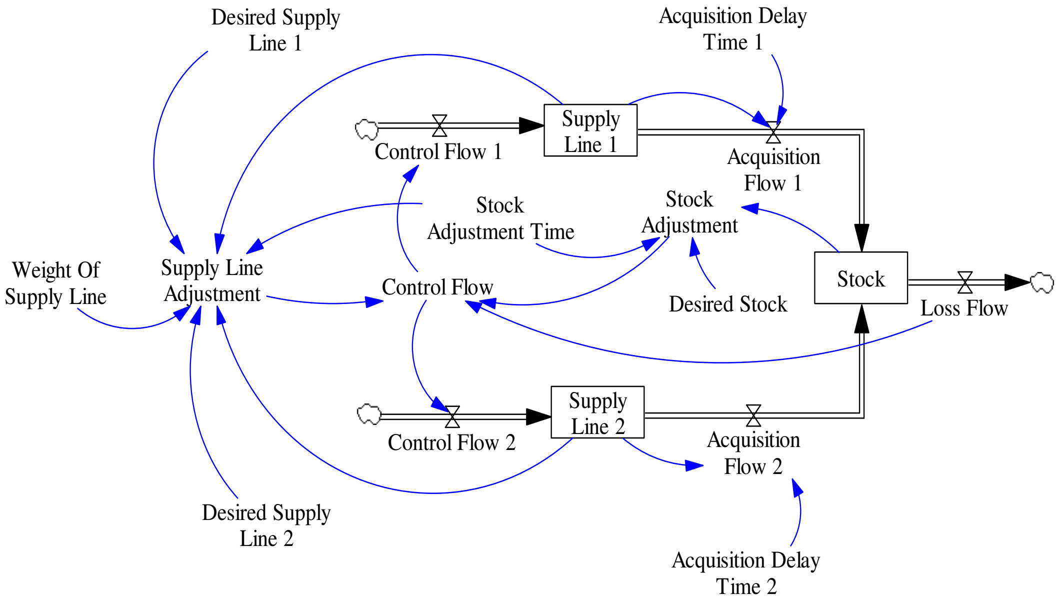

Figure 4.1. Stock-Flow Diagram of the Stock Management Task with 2 Supply Lines

If there is an error in determining the desired supply line values, there will be a steady-state error. Therefore, desired supply line values should correctly be selected. If the selected values are proper, the average value of *Stock* subtracted from its desired level will be equal to zero in the long run.

In order to balance the inflows to and outflow from the stock, the total average acquisition flow should be equal to the average loss flow:

$$\sum_{i=1}^{n} E[Acquisition\ Flow_i] = E[Loss\ Flow] \tag{4.2}$$

Average value of an inflow (i.e. control flow) attached to a supply line should balance the outflow (i.e. acquisition flow) from that supply line. This is needed to maintain each supply line around its desired value.

$$E[Control\ Flow_i] = E[Acquisition\ Flow_i] \qquad for \quad i = 1, 2, \cdots, n \tag{4.3}$$

Equations 4.2 and 4.3 yield Equation 4.4:

$$\sum_{i=1}^{n} E[\mathit{Control\ Flow}_i] = E[\mathit{Loss\ Flow}] \tag{4.4}$$

According to Equation 4.4, the total average control flow should also be equal to the average loss flow in the long run. *Desired Supply Line* values should be selected so as to satisfy equations 4.2 and 4.4.

It is known that the expected outflow from a supply line (i.e. expected acquisition flow) is equal to the average value of the supply line (i.e. desired supply line) divided by the delay time of that supply line (i.e. acquisition delay time).

$$E[\mathit{Acquisition\ Flow}_i] = \frac{\mathit{Desired\ Supply\ Line}_i}{\mathit{Aquisition\ Delay\ Time}_i} \tag{4.5}$$

Equations 4.2 and 4.5 yield Equation 4.6:

$$E[\mathit{Control\ Flow}_i] = \frac{\mathit{Desired\ Supply\ Line}_i}{\mathit{Aquisition\ Delay\ Time}_i} \tag{4.6}$$

From Equation 4.6, Equation 4.7 can be obtained:

$$\mathit{Desired\ Supply\ Line}_i = \mathit{Aquisition\ Delay\ Time}_i \times E[\mathit{Control\ Flow}_i] \tag{4.7}$$

The expected value of a control flow can be obtained using the priority assigned to the related supplier, and the probability distribution function of the loss flow. Once the expected control flow values are obtained, *Desired Supply Line* values can be obtained using Equation 4.7. Although Equation 4.7 is always valid, its application may not be that straightforward due to the difficulties in obtaining expected *Control Flow* values.

4.2. Multiple Supplier Examples

In this part, we give the applications of proposed *Desired Supply Line* calculation method for constant *Loss Flow* case and stochastic *Loss Flow* with a known stationary mean case. The distribution of *Loss Flow* (i.e. demand), supplier capacity limitations, and supplier priorities are the factors to be considered in control decisions. Handling stochastic demand is more problematic than handling constant demand in supply chain management (Nahmias, 2009; Jokar and Sajadieh, 2008, Schmitt, 2007). One other important concern in supply chain management is the capacity of suppliers. Production and shipment capacity constraints of suppliers lead to more oscillatory stock behaviors (Goncalves and Arango, 2010; Minner, 2003; Schmitt, 2007; Springer and Kim, 2010). The following control flow equation is used in both of the examples:

$$Total\ Control\ Flow = \begin{pmatrix} Expected\ Loss\ Flow + Stock\ Adjustment \\ + Supply\ Line\ Adjustment \end{pmatrix} \tag{4.8}$$

Note that, our examples assume three-supplier stock management system. The following individual orders to the three suppliers are calculated as given below. The priority of a supplier is represented by the index assigned to that supplier (low index represents high priority level).

$$\begin{pmatrix} Control \\ Flow_1 \end{pmatrix} = \begin{cases} Total\ Control\ Flow, & Total\ Control\ Flow \leq \begin{pmatrix} Capacity\ of \\ Supplier_1 \end{pmatrix} \\ Capacity\ of\ Supplier_1, & Total\ Control\ Flow \geq \begin{pmatrix} Capacity\ of \\ Supplier_1 \end{pmatrix} \end{cases} \tag{4.9}$$

$$\begin{pmatrix} Control \\ Flow_2 \end{pmatrix} = \begin{cases} \max\left(\begin{pmatrix} Total\ Control \\ Flow \\ - \\ Capacity\ of \\ Supplier_1 \end{pmatrix}, 0\right), & \begin{pmatrix} Total \\ Control \\ Flow \end{pmatrix} \leq \begin{pmatrix} Capacity\ of \\ Supplier_1 \\ + \\ Capacity\ of \\ Suppplier_2 \end{pmatrix} \\ Capacity\ of\ Supplier_2, & \begin{pmatrix} Total \\ Control \\ Flow \end{pmatrix} \geq \begin{pmatrix} Capacity\ of \\ Supplier_1 \\ + \\ Capacity\ of \\ Suppplier_2 \end{pmatrix} \end{cases} \tag{4.10}$$

$$\begin{pmatrix} Control \\ Flow_3 \end{pmatrix} = \max\left(Total\ Control\ Flow - \begin{pmatrix} Capacity\ of\ Supplier_1 \\ + \\ Capacity\ of\ Supplier_2 \end{pmatrix}, 0 \right) \tag{4.11}$$

4.2.1. Three-Supplier System with a Constant Loss Flow

In a single-supplier system, *Desired Supply Line* is calculated by using Equation 4.1 when *Loss Flow* is constant. Desired acquisition rate of the supply line is equal to *Loss Flow* given that supply line reaches its desired level.

In a multi-supplier system, desired acquisition rate of each supplier is selected by the decision maker depending on their priority levels and production/shipment capacities. The sum of the desired acquisition rates must be equal to *Loss Flow* in order to prevent a steady-state error. To calculate *Desired Supply Line* of a supplier, desired acquisition rate of that supplier must be multiplied by *Acquisition Delay Time* of the same supplier.

Let's assume there is a three-supplier system. The stock to be managed has a constant *Loss Flow* equal to 60. Acquisition delay times of the suppliers are 8, 12, and 16 in order. Target level of *Stock* is 0. The decision maker wants to receive 28 units from the first supplier, 22 units from the second supplier and 10 units from the third supplier. The desired value of each supply line is found by Equation 4.7. So, desired values of supply lines become 224, 264 and 160 in order. Notice that desired acquisition rate of a supplier must also be equal to expected control flow of that supplier for supply line stability.

As it can be seen from Figure 4.2, *Stock* stays on its desired level when *Stock* and its supply lines start at their desired levels. It is also observed from Figure 4.3 that even though *Stock* does not start at its desired level (starts at 250), both *Stock* and its supply lines seek their desired levels.

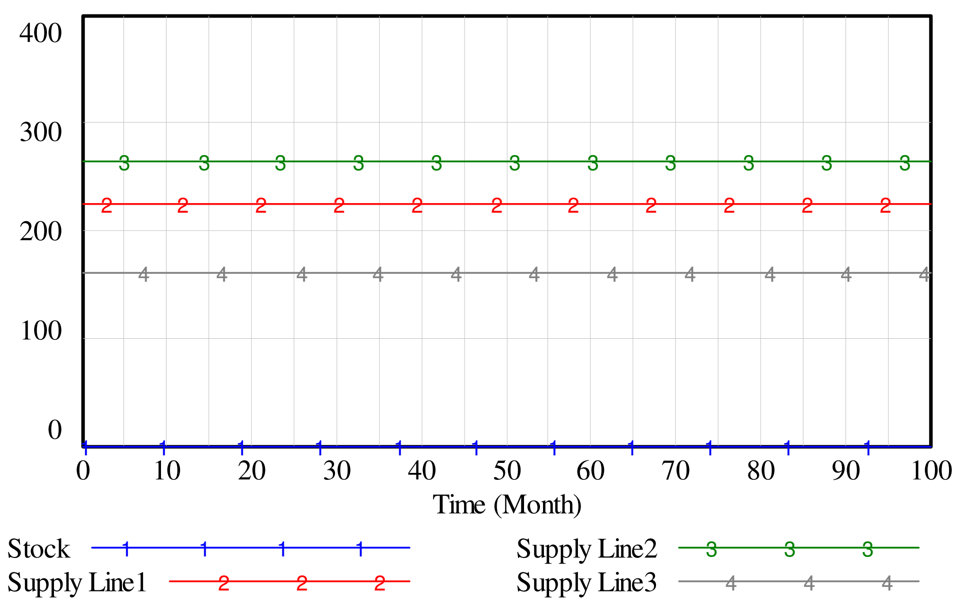

Figure 4.2. Stock and Supply Line Behaviors in a Three-Supplier System when *Loss Flow* is Constant

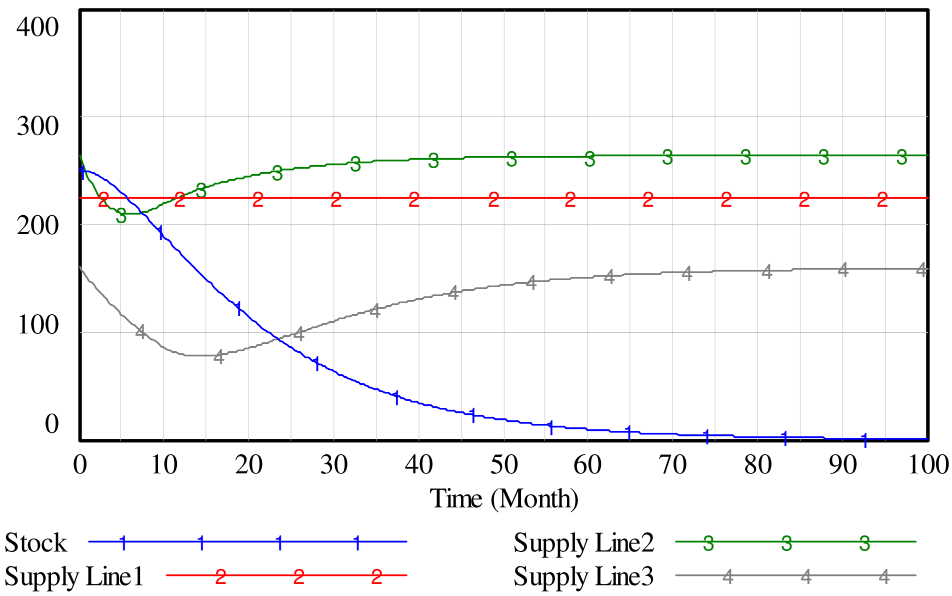

Figure 4.3. Stock and Supply Line Behaviors in a Three-Supplier System when *Loss Flow* is Constant and *Stock* does not start at its Desired Level

4.2.2. Three-Supplier System with a Stochastic Loss Flow

We have a three-supplier stock management model which has a stochastic *Loss Flow*. *Loss Flow* has a normal probability distribution with mean 60 and standard deviation 12. Simulation runs are obtained in discrete time; unity is used as the simulation time step. In this example, *Stock Adjustment Time* and *Weight of Supply Line* are taken as one. Under these assumptions, the distribution of *Control Flow* is equal to the distribution of *Loss Flow* (see Appendix B). In our model, the first supplier has priority over the second supplier and the second supplier has priority over the third. First and second suppliers have limited shipment, their capacity limits are 40 and 25 in order. The decision maker gives the orders up to 40 from the first supplier, orders between 40 and 65 from the second supplier, and orders above 65 from the second supplier. If order exceeds 40, first supplier provides 40 units and, if order exceeds 65, second supplier provides 25 units while first supplier still provides 40 units. *Desired Supply Line* depends on desired acquisition rate and acquisition lead time. Desired acquisition rates do not depend on acquisition lead time or the order of the supply line. However, they are affected by the capacity limitations of the suppliers. The upper limit of *Control Flow* of a supplier is its production/shipment capacity.

$$f(x) = \frac{1}{\sigma \times \sqrt{2 \times \pi}} \times e^{-\frac{1}{2} \times \left(\frac{x-\mu}{\sigma}\right)} \tag{4.12}$$

$$E[Control\ Flow_1] = \int_{-\infty}^{40} x \times f(x) \times dx + \int_{40}^{+\infty} 40 \times f(x) \times dx \tag{4.13}$$

$$E[Control\ Flow_2] = \int_{40}^{65} (x-40) \times f(x) \times dx + \int_{65}^{+\infty} 25 \times f(x) \times dx \tag{4.14}$$

$$E[Control\ Flow_3] = \int_{65}^{+\infty} (x-65) \times f(x) \times dx \tag{4.15}$$

Equation 4.12 shows the probability distribution function of normal distribution. Equations 4.13, 4.14, and 4.15 are valid because the distribution of *Control Flow* is equal to the distribution of *Loss Flow* (see Appendix B). According to equations 4.13, 4.14, and 4.15, desired acquisition rates are consecutively equal to 39.76207, 17.54096, and 2.696963. *Desired Supply Line* values become 318.0966, 210.4915, and 43.15141 consecutively for the first, second, and third suppliers (see Equation 4.7). If *.Desired Supply Line* values were calculated assuming constant *Loss Flow* (i.e. equal to mean of *Loss Flow* which is 60), they would be 320, 240, and 0 instead. This would lead to steady state error causing higher penalties.

In Figure 4.4, “DSL” on the x-axis corresponds to our base run which uses the calculated *Desired Supply Line* values of 318.0966, 210.4915, and 43.15141. Penalty values are generated by using equations 3.18 and 3.19 and the average of five different seeds is taken. The length of the simulations is 250. There are three lines in Figure 4.4 for the three suppliers. As one moves to the right on a line, *Desired Supply Line* value corresponding to that line increases while the other two *Desired Supply Line* values remain at their base levels. As one moves to the left on a line, *Desired Supply Line* value corresponding to that line decreases while the other two *Desired Supply Line* values remain at their base levels. An increase or a decrease in the proposed *Desired Supply Line* values results in an increase in the total penalties according to Figure 4.4. These results approve the appropriateness of our desired supply line value calculation method.

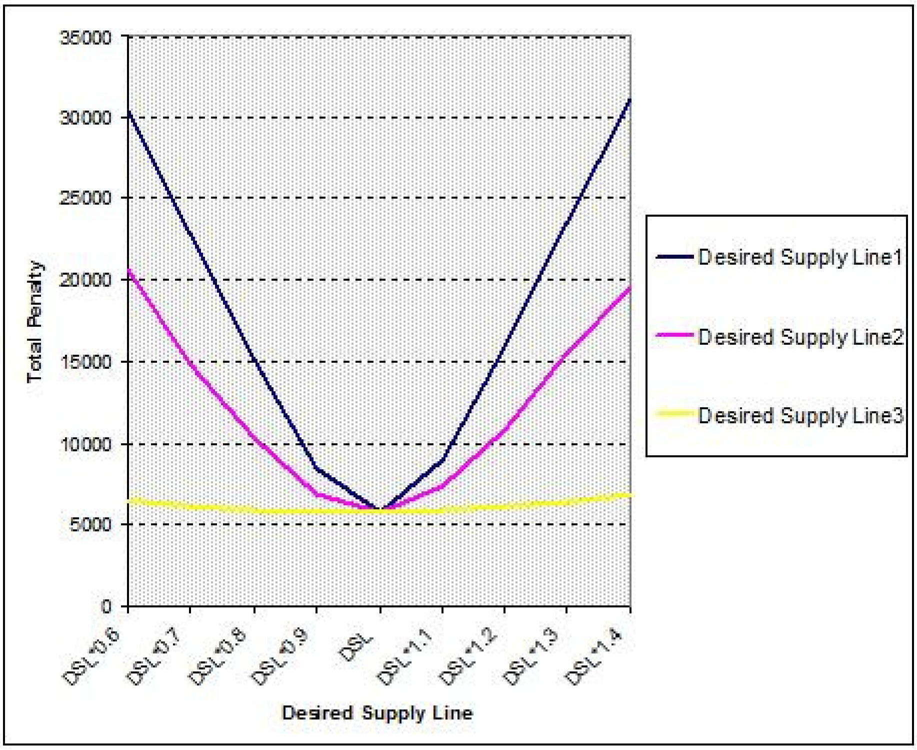

Figure 4.4. *Total Penalty* vs. *Desired Supply Line* Values

5. CONCLUSION

It is known that the presence of a supply line delay is one of the main reasons for the difficulty faced in managing a stock. Therefore, eliminating the delay or decreasing its duration (*Acquisition Delay Time*) should be considered in order to obtain a less complex stock management task (Diehl and Sterman, 1995; Paich and Sterman, 1993; Yasarcan, 2010; Yasarcan and Barlas, 2005b). One should also keep in mind that, eliminating or decreasing *Acquisition Delay Time* may not be practically possible or the associated costs may not be justifiable. In our work, *Acquisition Delay Time* and order of the delay structure are assumed to be given.

A change in Acquisition Delay Time and/or Stock Adjustment Time changes the stock dynamics, but if their ratio is kept constant, this change will have no effect on the nature of the stock behavior (no oscillations, stable oscillations, or unstable oscillations). Therefore, in order to simplify the analysis, we introduced a new parameter called *Relative Aggressiveness* which is equal to *Acquisition Delay Time* divided by *Stock Adjustment Time*. Instead of examining the effect of both the *Acquisition Delay Time* and *Stock Adjustment Time* on stock behavior, we examined the effect of *Relative Aggressiveness*. Introducing *Relative Aggressiveness* reduced our search space by one dimension.

Weight of Supply Line, *Relative Aggressiveness*, and the order of the delay structure determine the nature of the stock behavior and, thus, the associated penalties or costs. One's effect on the dynamics cannot be separated from the other because their interaction also has a significant effect on the stock behavior. Thus, the values for *Weight of Supply Line* and *Relative Aggressiveness* should be chosen considering the interaction between the two for a given order of the delay structure.

There is no theoretical upper limit for the optimum value of *Relative Aggressiveness* for a well selected *Weight of Supply Line* value (see figures 3.5 and 3.6). This implies that as *Stock Adjustment Time* decreases, *Total Penalty* also decreases once *Weight of Supply Line* is adjusted accordingly (see Equation 3.17 for *Stock Adjustment Time* and *Relative Aggressiveness* relationship). However, a very low *Stock Adjustment Time* value may create instabilities in a real life stock management system due to imperfections such as errors in parameter estimates, an unreliable supplier, delays or errors caused by decision making or measurement processes (Yasarcan, 2011; Yasarcan and Barlas, 2005b). An increase in *Stock Adjustment Time* value improves the stability of a system, but a high *Stock Adjustment Time* value (i.e. a low *Relative Aggressiveness*) makes the stock less responsive (i.e. stock approaches to its desired level relatively slow), which generates unnecessarily high penalties. We suggest setting *Relative Aggressiveness* as 4 balancing the two concerns (i.e. unresponsiveness and instability issues) assuming that *Weight of Supply Line* is chosen appropriately (see figures 3.5 and 3.6). This suggestion yields Equation 5.1. After estimating *Acquisition Delay Time*, Equation 5.1 can be used as a rule of thumb in setting *Stock Adjustment Time*. In other words, our suggestion to managers is that they should aim to close the gap between the actual and desired levels of *Stock* in a time period equal to one quarter of the delay duration.

$$Stock\ Adjustment\ Time = \frac{Acquisition\ Delay\ Time}{4} \quad [item/time] \tag{5.1}$$

In a discrete time stock management task, Equation 5.2 should be used instead of Equation 5.1 because a *Stock Adjustment Time* value less than the unit time step, which is one for discrete time, will potentially create instabilities.

$$\begin{pmatrix} Stock \\ Adjustment \\ Time \end{pmatrix} = \left\{ \begin{array}{ll} 1, & Acquisition\ Delay\ Time \leq 4 \\ \dfrac{Acquisition\ Delay\ Time}{4}, & Acquisition\ Delay\ Time > 4 \end{array} \right\} \quad \left[\frac{item}{time}\right] \tag{5.2}$$

One may prefer to use some other value for *Relative Aggressiveness* rather than 4. Even in that case, *Stock Adjustment Time* should be chosen based on *Acquisition Delay Time*. After assigning *Relative Aggressiveness* 4 or another reasonable value, *Stock Adjustment Time* can be calculated using Equation 5.1 (*Relative Aggressiveness* = 4; continuous time), Equation 5.2 (*Relative Aggressiveness* = 4; discrete time), Equation 3.17 (for any value of *Relative Aggressiveness*; continuous time), and Equation 5.3 (for any value of *Relative Aggressiveness*; discrete time).

$$\begin{pmatrix} Stock \\ Adjustment \\ Time \end{pmatrix} = \begin{cases} 1, & \begin{pmatrix} Acquisition \\ Delay\ Time \end{pmatrix} \leq \begin{pmatrix} Relative \\ Aggressiveness \end{pmatrix} \\ \dfrac{Acquisition\ Delay\ Time}{Relative\ Aggressiveness}, & \begin{pmatrix} Acquisition \\ Delay\ Time \end{pmatrix} > \begin{pmatrix} Relative \\ Aggressiveness \end{pmatrix} \end{cases} \quad \left[\frac{item}{time}\right] \tag{5.3}$$

After assigning a value to *Relative Aggressiveness*, Table 3.1 can be used for selecting a good value for *Weight of Supply Line*. To have an idea about how *Total Penalty* changes with respect to *Weight of Supply Line*, one can either examine Figure 3.7, which is obtained by setting *Relative Aggressiveness* equal to 4, or the contour plots in figures 3.5 and 3.6. The contour plots in figures 3.5 and 3.6 suggest that a *Relative Aggressiveness* value less than 1 (i.e. a *Stock Adjustment Time* value bigger than *Acquisition Delay Time*) should be avoided as it creates high *Total Penalty* values. A low *Weight of Supply Line* value should also be avoided. Although it is not suggested, if for some reason, a decision maker prefers using a low *Weight of Supply Line* value, she should select *Relative Aggressiveness* value with extreme caution because a low *Weight of Supply Line* value can potentially generate unstable oscillations resulting in high penalties. Another interesting result worth to be mentioned is that, even though unity is a value that guarantees non-oscillatory stock behavior, it is not the optimum *Weight of Supply Line* value for different cases with different delay orders and *Relative Aggressiveness* values. For example, for a first order supply line delay structure, the optimum *Weight of Supply Line* value is 0.425 when *Stock Adjustment Time* is a quarter of *Acquisition Delay Time* (i.e. *Relative Aggressiveness* is equal to 4).

If the values of the two decision parameters *Weight of Supply Line* and *Stock Adjustment Time* are chosen as suggested in this paper, a significant improvement in managerial decision making process will be attained for the stock management task. As a future study, we are planning to develop interventions for training managers based on the findings reported in this paper. We are also planning to test the suggested rule of thumb of setting *Stock Adjustment Time* to a quarter of *Acquisition Delay Time* in a simulated supply chain environment.

A wrongly calculated *Desired Supply Line* value leads to a steady-state error preventing stock approach its goal. Therefore, desired supply line values should correctly be selected. In Chapter 4, the calculation of *Desired Supply Line* values in multi-supplier systems is examined. Using multiple suppliers instead of a single supplier reduces the procurement risks in stock management. However, determining *Desired Supply Line* values in a multi-supplier system is not that straightforward compared to a single-supplier system.

In steady state, inflow (*Control Flow*) to a supply line has to be equal to the outflow (*Acquisition Flow*) from that supply line. Also, the total inflow (sum of all acquisition flows) to a stock has to be equal to the outflow (*Loss Flow*) from that stock. Note that, outflow from a supply line is, at the same time, an inflow to the corresponding stock. Eventually, this brings the deduction that the sum of all inflows to the supply lines in a stock management system (i.e. control flows) has to be equal to the outflow (i.e. *Loss Flow*) from the main stock of that system. Therefore, the selection of *Desired Supply Line* values must ensure that different supply lines in total produce the total desired acquisition rate. In Chapter 4, we give a general approach in obtaining proper *Desired Supply Line* values in a multi-supplier stock management system. The desired values obtained by using this approach make the average value of *Stock* subtracted from its desired level equal to zero in the long run.

According to the general approach in determining the *Desired Supply Line* values in a multi-supplier stock management system, once the expected control flow values are obtained, *Desired Supply Line* values can be obtained using Equation 4.7. Although Equation 4.7 is always valid, its application may not be that straightforward due to the difficulties in obtaining expected *Control Flow* values. In this study, this approach is applied to two cases: one under constant *Loss Flow* assumption and the other one under stochastic *Loss Flow* (normally distributed with known mean and variance) assumption. As a continuation of this study, we are planning first to extend the application of this approach to a case under stochastic *Loss Flow* (normally distributed with unknown mean and variance) assumption with exponential smoothing heuristic used in expectation formation. Secondly, the generality of the results obtained from the first extension of the study will be discussed for other *Loss Flow* distributions.

APPENDIX A: MATHEMATICAL PROOF FOR THE EFFECT OF THE RATIO BEETWEEN ACQUISITION DELAY TIME AND STOCK ADJUSTMENT TIME

Stock equations of the model seen in Figure 3.1 are presented below:

$$\dot{Stock} = Acquisition\,Flow - Loss\,Flow \tag{A.1}$$

$$\dot{Stock} = \frac{Supply\,Line}{ADT} - Loss\,Flow \tag{A.2}$$

$$\dot{Supply\,Line} = Control\,Flow - Acquisition\,Flow \tag{A.3}$$

$$\dot{Supply\,Line} = \begin{pmatrix} Stock\ Adjustment \\ + \\ Supply\,Line\ Adjustment \\ + \\ Loss\,Flow \end{pmatrix} - \frac{Supply\,Line}{ADT} \tag{A.4}$$

$$\dot{Supply\,Line} = \begin{pmatrix} \dfrac{Desired\,Stock - Stock}{SAT} \\ + \\ WSL \times \dfrac{Desired\,Supply\,Line - Supply\,Line}{SAT} \\ + \\ Loss\,Flow \end{pmatrix} - \frac{Supply\,Line}{ADT} \tag{A.5}$$

The matrix notation of stock equations is seen in Equation A.6.

$$\begin{pmatrix} \dot{Stock} \\ \dot{Supply\,Line} \end{pmatrix} = \mathrm{A} \times \begin{pmatrix} Stock \\ Supply\,Line \end{pmatrix} \quad (A.6)$$

The coefficient matrix A is presented in Equation A.7.

$$\mathrm{A} = \begin{pmatrix} 0 & \frac{1}{ADT} \\ -\frac{1}{SAT} & -\frac{1}{ADT} - \frac{WSL}{SAT} \end{pmatrix} \quad (A.7)$$

The characteristic determinant of coefficient matrix A can be seen in Equation A.8.

$$|\mathrm{A} - \lambda \cdot \mathrm{I}| = \begin{pmatrix} -\lambda & \frac{1}{ADT} \\ -\frac{1}{SAT} & -\lambda - \frac{1}{ADT} - \frac{WSL}{SAT} \end{pmatrix} \quad (A.8)$$

The characteristic equation is seen in Equation A.9.

$$\lambda^2 + \lambda \times \left(\frac{WSL}{SAT} + \frac{1}{ADT} \right) + \frac{1}{ADT \times SAT} = 0 \quad (A.9)$$

The roots of the characteristic equation are shown in Equation A.10.

$$\lambda_{1,2} = \frac{-\left(\frac{WSL}{SAT} + \frac{1}{ADT} \right) \pm \sqrt{\left(\frac{WSL}{SAT} + \frac{1}{ADT} \right)^2 - \frac{4}{ADT \times SAT}}}{2} \quad (A.10)$$

The condition that the roots of the characteristic equation have imaginary parts is presented in Equation A.11. When the roots of characteristic equation have imaginary parts, oscillatory behaviors occur in the stocks of the system.

$$\left(\frac{WSL}{SAT}+\frac{1}{ADT}\right)^2-\frac{4}{ADT\times SAT}<0 \quad \text{(A.11)}$$

Equation A.12 can also be written as equation A.12.

$$WSL^2\times\frac{ADT}{SAT}+2\times WSL+\frac{SAT}{ADT}<4 \quad \text{(A.12)}$$

It is observed from Equation A.12 that the existence of imaginary roots, thus oscillations, depends on *Weight of Supply Line* and the ratio between *Acquisition Delay Time* and *Stock Adjustment Time*. Therefore, we introduce *Relative Aggressiveness*, which is *Acquisition Delay Time* divided by *Stock Adjustment Time*, into our analysis. So, equation A.12 can also be expressed as equation A.13.

$$WSL^2\times RA+2\times WSL+\frac{1}{RA}<4 \quad \text{(A.13)}$$

When *Weight of Supply Line* is equal to zero Equation A.13 becomes:

$$\frac{1}{RA}<4 \quad \text{(A.14)}$$

According to A.14, goal seeking behavior is observed when *Relative Aggressiveness* is smaller than 0.25 and damping oscillations are observed when it is greater than 0.25.

APPENDIX B: PROOF FOR THE EQUIVALENCE OF THE PROBABILITY DISTRIBUTIONS OF CONTROL FLOW AND LOSS FLOW

Stock Adjustment Time and *Weight of Supply Line* are both chosen as 1. With these settings, two consecutive control flows are calculated by equations B.15 and B.16.

$$Control\,Flow_i = Expected\,Loss\,Flow + \begin{pmatrix} Desired\,Stock \\ - \\ Stock_i \end{pmatrix} + \begin{pmatrix} Desired\,Supply\,Line \\ - \\ SupplyLine_i \end{pmatrix} \quad \text{(B.15)}$$

$$Control\,Flow_{i+1} = Expected\,Loss\,Flow + \begin{pmatrix} Desired\,Stock \\ - \\ Stock_{i+1} \end{pmatrix} + \begin{pmatrix} Desired\,Supply\,Line \\ - \\ SupplyLine_{i+1} \end{pmatrix} \quad \text{(B.16)}$$

Stock formulation is seen in Equation B.17 and Supply Line formulation is seen in Equation B.18 with these settings.

$$Stock_{i+1} = Stock_i + Acquisition\,Flow_i - Loss\,Flow_i \quad \text{(B.17)}$$

$$Supply\,Line_{i+1} = Supply\,Line_i + Control\,Flow_i - Acquisition\,Flow_i \quad \text{(B.18)}$$

Difference of two consecutive control flows is shown in Equation B.19.

$$Control\,Flow_{i+1} - Control\,Flow_i = Stock_i - Stock_{i+1} + Supply\,Line_i - Supply\,Line_{i+1} \quad \text{(B.19)}$$

When equations B.17 and B.18 are plugged in to Equation B.19, so Equation B.20 is obtained.

$$Control\,Flow_{i+1} - Control\,Flow_i = Loss\,Flow_i - Control\,Flow_i \quad \text{(B.20)}$$

Equation B.20 yields Equation B.21. Equation B.21 shows that *Control Flow* follows *Loss Flow* from 1 time unit behind with these settings. This means that their probability distributions are exactly the same. Therefore, our expected control flows can be calculated by using the probability distribution of *Loss Flow*.

$$Control\,Flow_{i+1} = Loss\,Flow_{i} \tag{B.21}$$

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